3/28/2011
3/27/2011
3/26/2011 RevB
1/2/2011 original

Derivation of the inverse kinematics

(calculation of wheel speeds and wheel angles)

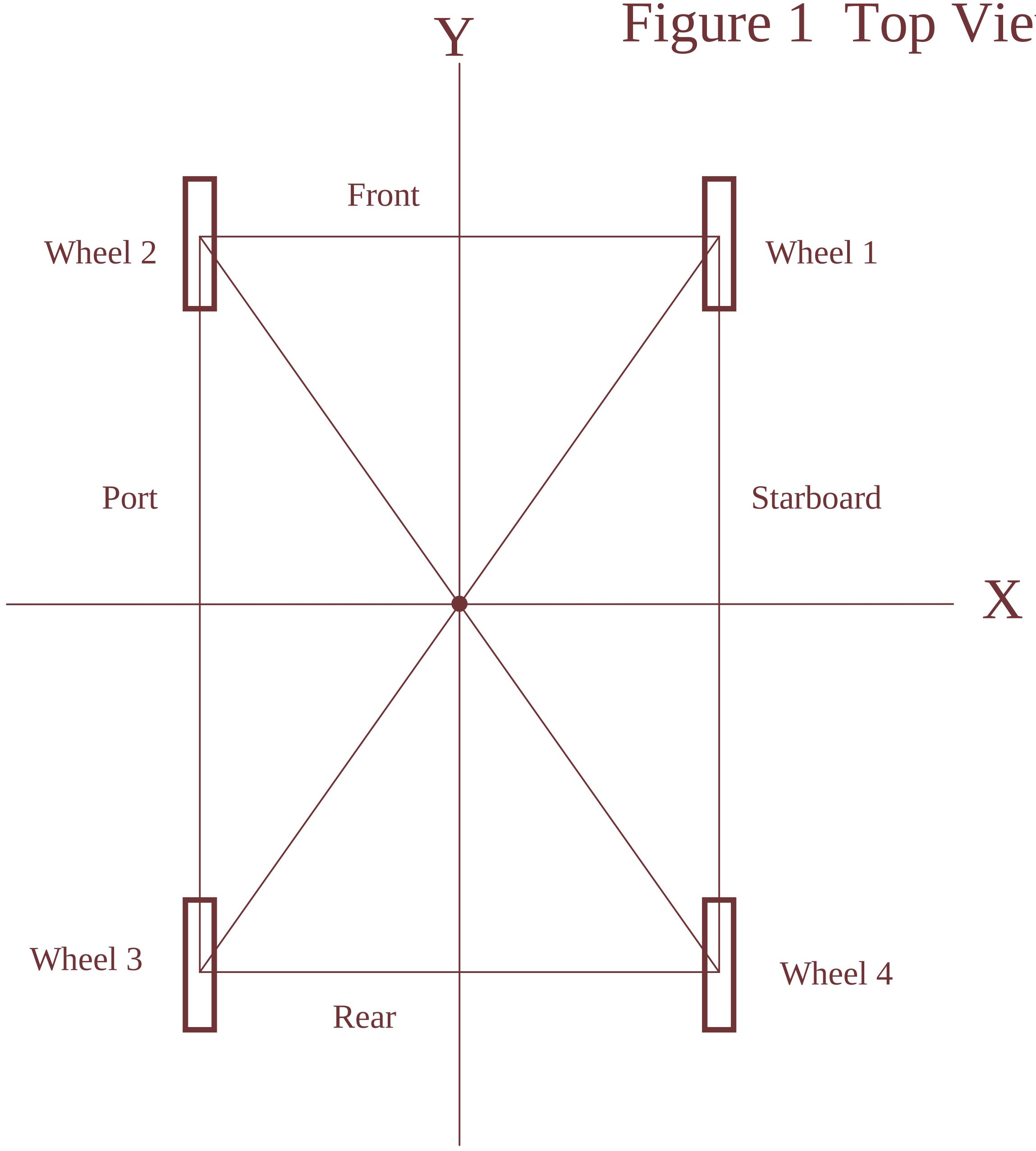
for three-degree-of-freedom control

of vehicle with four-wheel independent drive

and independent steering

(sometimes a.k.a. “Swerve" drive)

Figure 1 Top View

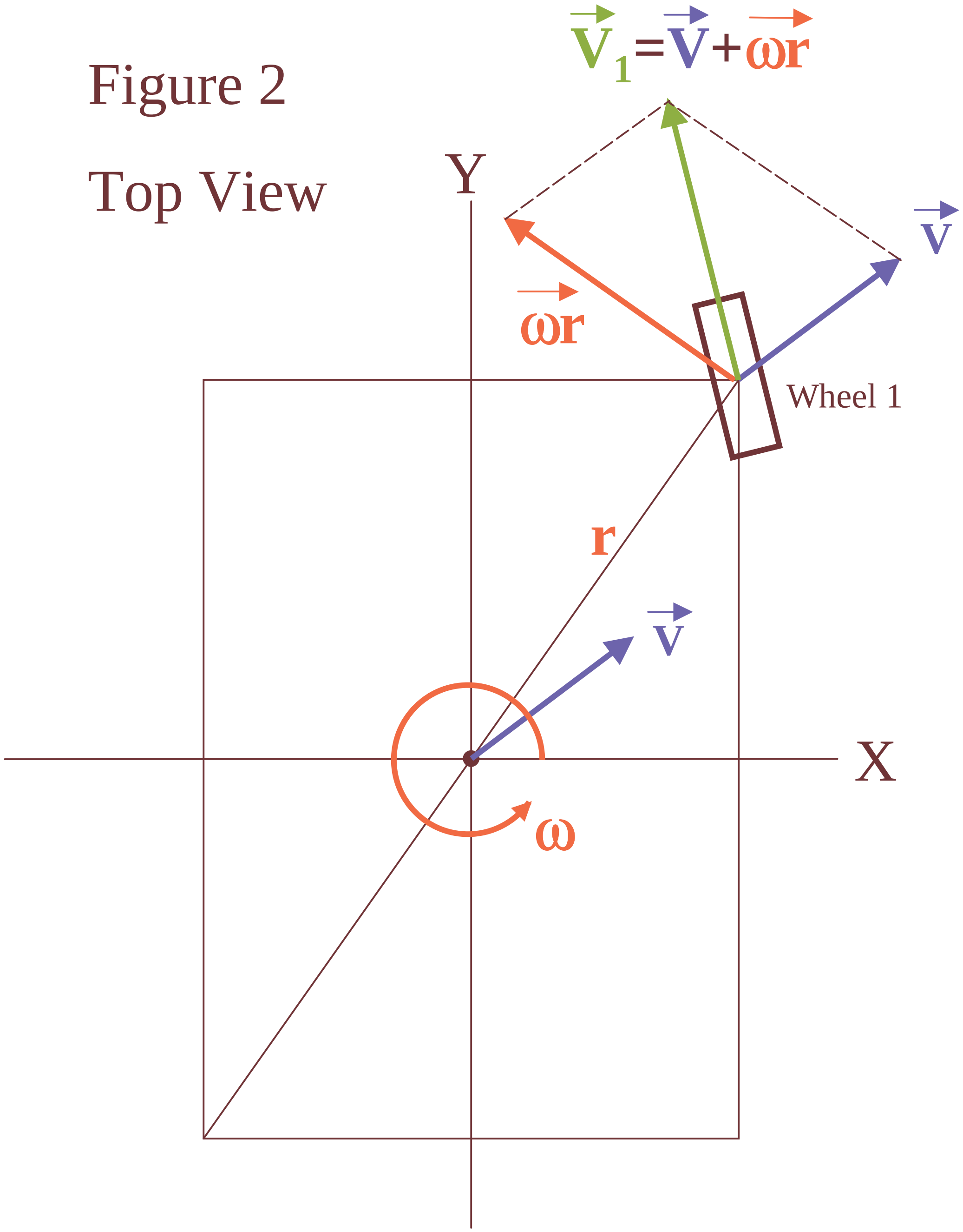


Each wheel independently driven and steered

360 degree wheel steering

Figure 2

Top View



Legend:

$\vec{V}$ Vehicle translation (fwd/rev plus strafe)

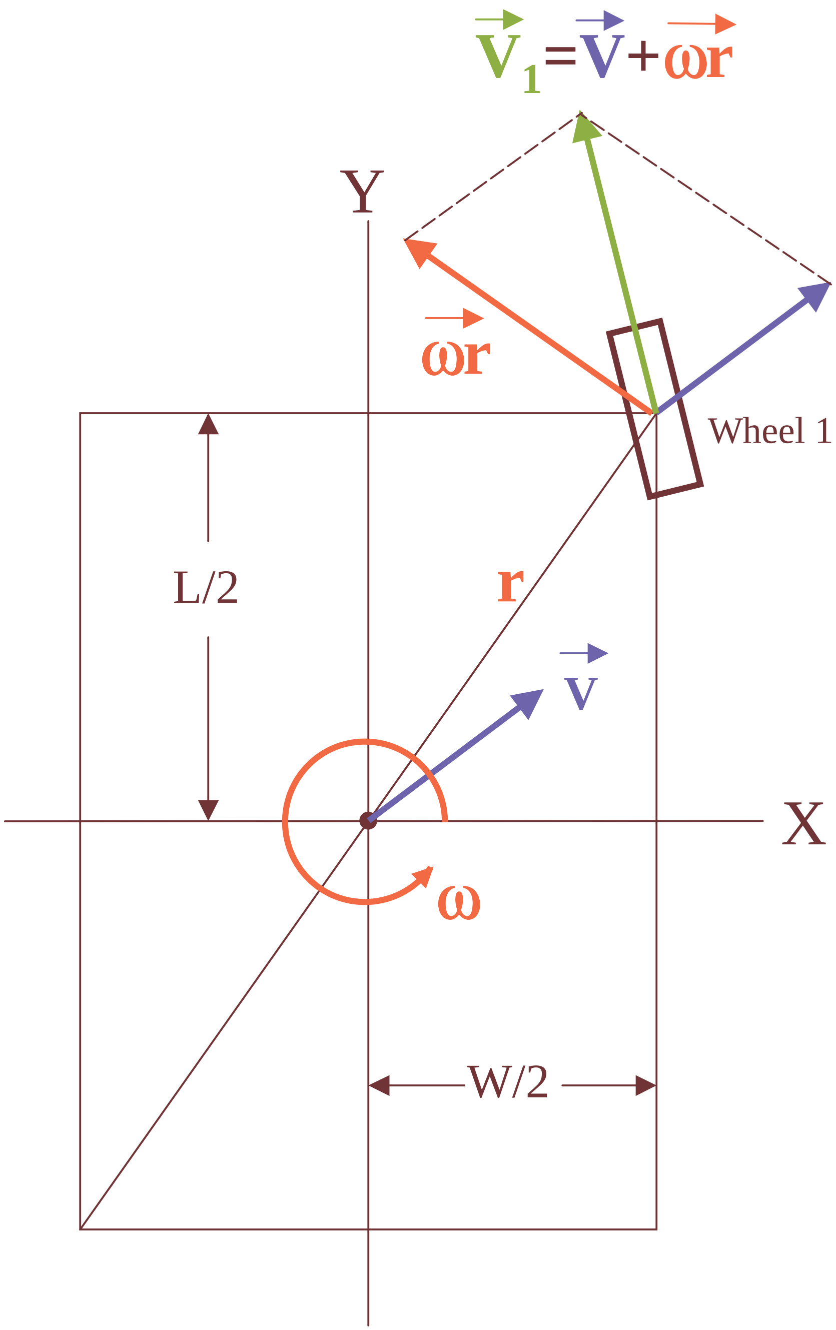
ω Vehicle rotation [1]

$\vec{V}_1$ Wheel#1 direction and velocity [2]

[1]omega is radians/sec and is positive clockwise. The example in this diagram is a negative (counterclockwise) vehicle rotation. The formulas in Figure 6 for the wheel steering will give clockwise angles in degrees.

[2]calculated separately for each of the 4 wheels from the vehicle translation and rotation

Figure 3 Top View



$$V_{1x} = V_x + (\omega r)_x = V_x + \omega L/2$$

$$V_{1y} = V_y + (\omega r)_y = V_y - \omega W/2$$

V_x **is the X component of** $\vec{V}$

V_y **is the Y component of** $\vec{V}$

$(\omega r)_x$ **is the X component of** $\vec{\omega r}$

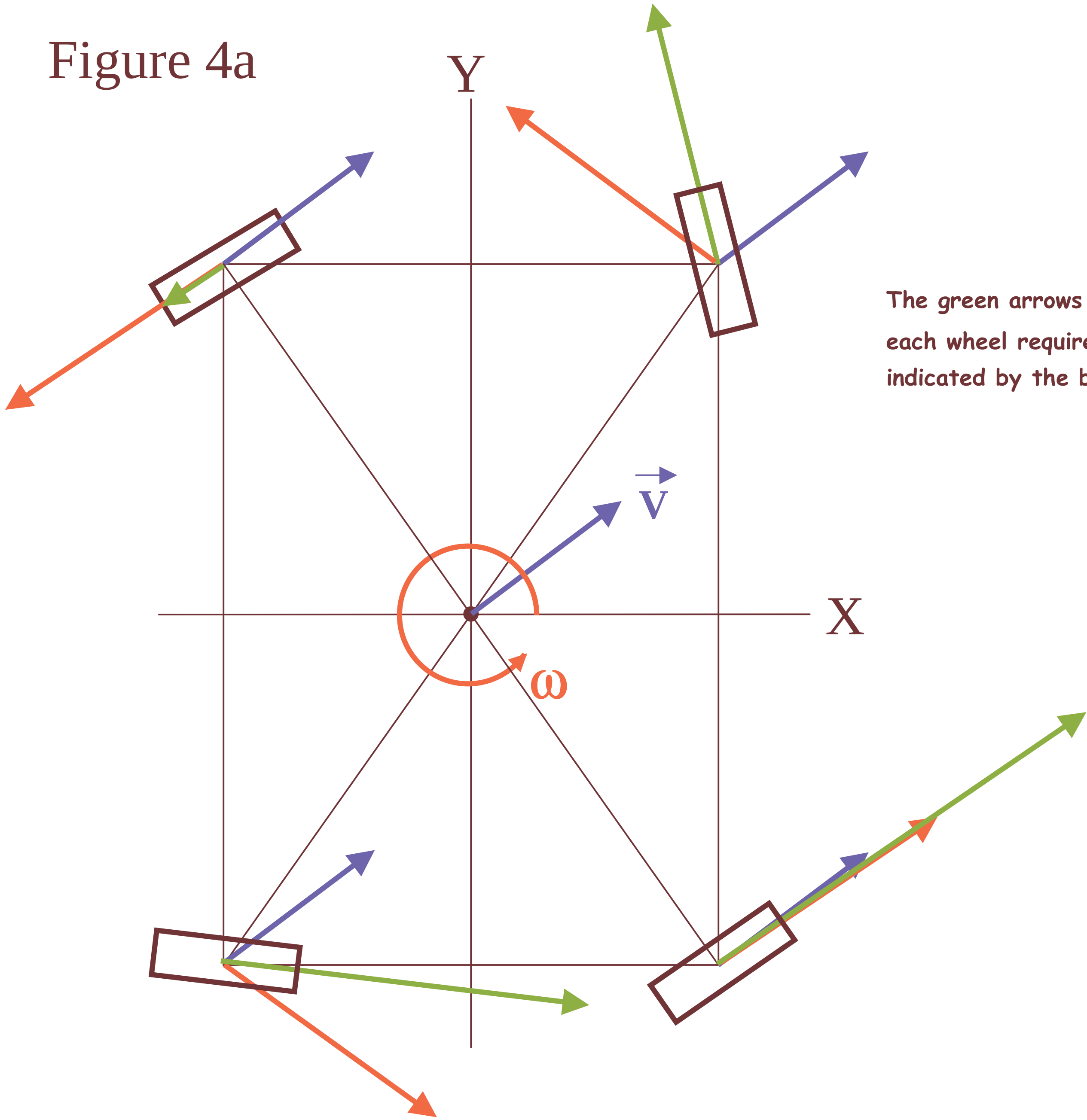
$(\omega r)_y$ **is the Y component of** $\vec{\omega r}$

L **is the wheelbase**

W **is the trackwidth**

r $= \mathrm{sqrt}(L^2+W^2)/2$

Figure 4a



The green arrows show the speed and direction of each wheel required to produce the vehicle motion indicated by the blue V and red ω

Figure 4b

Wheel 2

$$V_{2x} = V_x + (\omega r)_x = V_x + \omega L/2$$

$$V_{2y} = V_y + (\omega r)_y = V_y + \omega W/2$$

Wheel 1

$$V_{1x} = V_x + (\omega r)_x = V_x + \omega L/2$$

$$V_{1y} = V_y + (\omega r)_y = V_y - \omega W/2$$

Wheel 3

$$V_{3x} = V_x + (\omega r)_x = V_x - \omega L/2$$

$$V_{3y} = V_y + (\omega r)_y = V_y + \omega W/2$$

Wheel 4

$$V_{4x} = V_x + (\omega r)_x = V_x - \omega L/2$$

$$V_{4y} = V_y + (\omega r)_y = V_y - \omega W/2$$

Figure 5

Define A, B, C, & D as follows:

$$A = V_x - \omega L/2 \qquad B = V_x + \omega L/2$$

$$C = V_y - \omega W/2 \qquad D = V_y + \omega W/2$$

… and the chart from Figure 4b becomes:

Wheel 2 $V_{2x} = B$ $\quad V_{2y} = D$	**Wheel 1** $V_{1x} = B$ $\quad V_{1y} = C$
Wheel 3 $V_{3x} = A$ $\quad V_{3y} = D$	**Wheel 4** $V_{4x} = A$ $\quad V_{4y} = C$

Figure 6

Calculate the speed and angle of each wheel

Wheel 2

speed = $\text{sqrt}(B^2 + D^2)$

angle = atan2(B,D)*180/pi

Wheel 1

speed = $\text{sqrt}(B^2 + C^2)$

angle = atan2(B,C)*180/pi

Wheel 3

speed = $\text{sqrt}(A^2 + D^2)$

angle = atan2(A,D)*180/pi

Wheel 4

speed = $\text{sqrt}(A^2 + C^2)$

angle = atan2(A,C)*180/pi

1] Angles range from -180 to +180 degrees CW; zero is straight ahead

2] If, after calculating the 4 wheel speeds, any of them is greater than 1, then divide all the wheel speeds by the largest value.

3/28/2011 revision

1) Revised for CLOCKWISE ANGLES (for wheel steering θ and vehicle rotation ω)

2) Redefined L & W as full wheelbase and trackwidth

3) Vehicle rotate command is now in radians/sec instead of linear dimensions